1. Introduction to Time Series

A **time series** is a collection of observations recorded **sequentially over time**. Each data point in a time series is typically associated with a specific time stamp. These sequences allow us to analyze how a particular quantity evolves or fluctuates over time.

Examples of Time Series Data:

- **Stock prices** recorded every minute or daily.
- **Daily temperatures** in a city.
- **Monthly sales figures** for a product.=
- **ECG signals** captured every millisecond in a medical test.

The essential feature that distinguishes time series data from other types, such as **cross-sectional data**, is the **importance of time**. In cross-sectional data, observations are collected at the same point in time across different entities (e.g., income levels across individuals in a year), and the order of data doesn't matter. However, in time series, **the order of the data points is crucial** because each point depends, at least to some extent, on previous points.

Key Characteristics of Time Series

a) Temporal Ordering

- **Definition**: Time series data is arranged in chronological order. This means that each observation is connected to a specific time point (like a day, month, or year), and the sequence in which data appears is essential.
- **Example**: If you record the temperature at noon every day for a week, the data must be in order (Monday, Tuesday, …, Sunday) to correctly observe any trend or pattern.

b) Dependence

- **Definition**: Unlike in other types of data where observations are often assumed to be independent, in time series, the current observation is often **influenced by past values**.
- **Why It Matters**: This temporal dependency is what makes time series analysis unique. For example, today's temperature is likely related to yesterday's, and tomorrow's sales might depend on today's promotions.
- **Implication**: Standard statistical techniques that assume independence between observations are not suitable for time series unless the dependency is accounted for.

c) Trend, Seasonality, and Noise

Time series data often show one or more of the following components:

1. **Trend**:

    - A long-term increase or decrease in the data.
    - Example: Over several years, the average global temperature might show an upward trend.
2. **Seasonality**:
    - Repeating patterns or cycles over a fixed period, such as daily, weekly, or annually.
    - Example: Ice cream sales may peak every summer.
3. **Noise**:
    - Random variation in the data that does not follow a predictable pattern.
    - Example: Sudden drops in stock prices due to unexpected news.

These components are often combined in a time series and need to be identified and analyzed to make accurate forecasts or understand the behavior of the system over time.

2. Components of a Time Series

Time series data can be broken down into **four fundamental components** that help in understanding and analyzing its behavior over time. These components are essential for modeling and forecasting time series accurately.

1. Trend (T)

- **Definition**: The **trend** represents the **long-term direction** in which the data is moving over time.
- **Nature**: It can be **upward**, **downward**, or **flat** (no trend).
- **Examples**:
    - The steady growth in population over years.
    - Increasing stock prices over a decade.
    - Gradual decline in sales of DVDs due to digital streaming.

The trend is not about short-term fluctuations but a **persistent movement** over a longer period, and it can be linear or nonlinear.

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2. Seasonality (S)

- **Definition**: **Seasonality** refers to patterns that **repeat at regular time intervals**, typically due to external influences like climate, holidays, or events.

- **Time Frame**: Can be **daily, weekly, monthly, quarterly, or yearly**.
- **Examples**:
    - Increased retail sales during **December** due to holidays.
    - Higher electricity usage in **summer** due to air conditioning.
    - Daily spikes in website traffic during specific hours.

Seasonality is **predictable and periodic**.

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3. Cyclic (C)

- **Definition**: The **cyclic component** reflects **long-term fluctuations** that are **not fixed** in period but **follow some cycle**, often linked to economic or business conditions.
- **Duration**: Longer and more **variable** than seasonal effects.
- **Examples**:
    - Economic expansions and recessions.
    - Commodity price cycles influenced by supply and demand over years.

Cyclic patterns differ from seasonal patterns in that their **length and amplitude can change**, and they are **not calendar-based**.

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4. Irregular or Random (I)

- **Definition**: This is the **residual or noise** in the time series after trend, seasonality, and cyclic effects have been removed.
- **Characteristics**:
    - **Unpredictable**, **unsystematic**, and **short-term** fluctuations.
    - Often caused by **unexpected events** like natural disasters, strikes, or accidents.
- **Examples**:
    - A sudden spike in flight cancellations due to a volcanic eruption.
    - Random variations in sensor readings.

Irregular components cannot be modeled or predicted and are treated as **random noise**.

Time Series Models

To analyze a time series, we often represent it mathematically using one of the following models:

Additive Model

In the **additive model**, the time series is assumed to be the **sum** of its components:

Y(t)=T(t)+S(t)+C(t)+I(t)Y(t)

Suitable when the **magnitude of seasonal or cyclic changes does not depend on the level of the series**.

- Used when seasonal effects are **roughly constant** over time.

Example:
If sales increase by 200 units every December regardless of the overall sales level, this pattern is additive.

Multiplicative Model

In the **multiplicative model**, the time series is assumed to be the **product** of its components:

Y(t)=T(t)×S(t)×C(t)×I(t)

Suitable when the **seasonal or cyclic variation grows/shrinks proportionally** with the level of the trend.

- Used when fluctuations **increase with the overall value** of the time series.

Example:
If sales increase by **20% every December**, rather than by a fixed amount, this is multiplicative.

Comparison of Additive vs Multiplicative Models

Feature	**Additive Model**	**Multiplicative Model**
Seasonal effect	Constant over time	Varies with trend
Mathematical form	Y(t)=T+S+C+IY	Y(t)=T×S×C×I
When to use	When variation is stable	When variation grows with the data
Visual pattern	Flat seasonal variation	Widening/narrowing seasonal effects

🔢 Additive Model – Numerical Example

Given Data:

We observe monthly sales data over a few months and assume:

- Trend (T) = 100 units per month (linear upward trend)
- Seasonal (S) = December adds +200 units due to holiday sales
- Cyclic (C) = Small business cycle effect = +50 units
- Irregular (I) = Random irregularity (noise) = +10 units

We calculate sales for **December** using the **additive model**:

$$Y(t) = T(t) + S(t) + C(t) + I(t)$$

December:

- T(Dec) = 100
- S(Dec) = +200
- C(Dec) = +50
- I(Dec) = +10

$$Y(\text{Dec}) = 100 + 200 + 50 + 10 = \boxed{360 \text{ units}}$$

This shows that **seasonal effect is constant** (+200 units every December), regardless of trend level.

🔢 Multiplicative Model – Numerical Example

Given Data:

Assume a product's sales increase **20% in December** due to holidays. The components are:

- **Trend** (T) = 100 units
- **Seasonal** (S) = 1.20 (i.e., +20%)
- **Cyclic** (C) = 1.05 (i.e., +5%)
- **Irregular** (I) = 1.10 (i.e., +10%)

Using the multiplicative model:

$$Y(t) = T(t) \times S(t) \times C(t) \times I(t)$$

December:

$$Y(\text{Dec}) = 100 \times 1.20 \times 1.05 \times 1.10 = \boxed{138.6 \text{ units}}$$

This shows that **seasonal effect (20%) is proportional to the trend level**, so as the base sales increase, the December spike increases as well.

Summary Table:

Model Type	Equation	December Output	Seasonal Effect
Additive	Y = T + S + C + I	360 units	Constant (+200 units)
Multiplicative	Y = T × S × C × I	138.6 units	Proportional (×1.20)