

Kernel matrix computation

Step 1: Compute the Kernel Matrix

Each element $K(i, j)$ in the matrix is computed as:

$$K(x_i, x_j) = (x_i \cdot x_j + 1)^2$$

where $x_i \cdot x_j$ is the dot product.

Computing Pairwise Kernel Values:

$$K(1, 1) = [(1 \times 1 + 2 \times 2) + 1]^2 = (1 + 4 + 1)^2 = 6^2 = 36$$

$$K(1, 2) = [(1 \times 2 + 2 \times 3) + 1]^2 = (2 + 6 + 1)^2 = 9^2 = 81$$

$$K(1, 3) = [(1 \times 3 + 2 \times 3) + 1]^2 = (3 + 6 + 1)^2 = 10^2 = 100$$

$$K(1, 4) = [(1 \times 5 + 2 \times 1) + 1]^2 = (5 + 2 + 1)^2 = 8^2 = 64$$

Repeating for all points, the **Kernel Matrix** is:

compute all elements of the kernel matrix using the kernel function

$$K(x_i, x_j) = (x_i \cdot x_j + 1)^2$$

where $x_i \cdot x_j$ is the dot product. Let's go **step by step** for all points.

From the image, your data points seem to be:

$$x_1 = [1, 2], \quad x_2 = [2, 3], \quad x_3 = [3, 3], \quad x_4 = [5, 1]$$

Step 1: Compute pairwise kernel values

1. K(1,1)

$$K(1, 1) = [(1 \cdot 1 + 2 \cdot 2) + 1]^2 = (1 + 4 + 1)^2 = 6^2 = 36$$

2. K(1,2)

$$K(1, 2) = [(1 \cdot 2 + 2 \cdot 3) + 1]^2 = (2 + 6 + 1)^2 = 9^2 = 81$$

3. K(1,3)

$$K(1, 3) = [(1 \cdot 3 + 2 \cdot 3) + 1]^2 = (3 + 6 + 1)^2 = 10^2 = 100$$

4. K(1,4)

$$K(1, 4) = [(1 \cdot 5 + 2 \cdot 1) + 1]^2 = (5 + 2 + 1)^2 = 8^2 = 64$$

5. $K(2,1)$ (symmetric, same as $K(1,2)$)

$$K(2, 1) = 81$$

6. $K(2,2)$

$$K(2, 2) = [(2 \cdot 2 + 3 \cdot 3) + 1]^2 = (4 + 9 + 1)^2 = 14^2 = 196$$

7. $K(2,3)$

$$K(2, 3) = [(2 \cdot 3 + 3 \cdot 3) + 1]^2 = (6 + 9 + 1)^2 = 16^2 = 256$$

8. $K(2,4)$

$$K(2, 4) = [(2 \cdot 5 + 3 \cdot 1) + 1]^2 = (10 + 3 + 1)^2 = 14^2 = 196$$

9. $K(3,1) = 100$ (symmetric)

10. $K(3,2) = 256$ (symmetric)

11. $K(3,3)$

$$K(3, 3) = [(3 \cdot 3 + 3 \cdot 3) + 1]^2 = (9 + 9 + 1)^2 = 19^2 = 361$$

12. $K(3,4)$

$$K(3, 4) = [(3 \cdot 5 + 3 \cdot 1) + 1]^2 = (15 + 3 + 1)^2 = 19^2 = 361$$

13. $K(4,1) = 64$ (symmetric)

14. $K(4,2) = 196$ (symmetric)

15. $K(4,3) = 361$ (symmetric)

16. $K(4,4)$

$$K(4, 4) = [(5 \cdot 5 + 1 \cdot 1) + 1]^2 = (25 + 1 + 1)^2 = 27^2 = 729$$

Step 2: Kernel Matrix

$$K = \begin{bmatrix} 36 & 81 & 100 & 64 \\ 81 & 196 & 256 & 196 \\ 100 & 256 & 361 & 361 \\ 64 & 196 & 361 & 729 \end{bmatrix}$$

