PLS with One Independent Variable

👉 If we only have **one predictor** X, PLS reduces to **simple linear regression**, because there is only one possible latent component (the variable itself).

- The algorithm will essentially give the same result as Ordinary Least Squares (OLS).
- But we still can demonstrate it step by step in a small example.

🔷 Example Exercise:

We want to predict Y (output) from X (single predictor).

Observation	X	Y
1	1	2
2	2	4
3	3	5
4	4	4
5	5	5

We fit **PLS regression with one component**.

Step 1: Standardize X and Y (mean-centering)

$$\bar{X} = \frac{1+2+3+4+5}{5} = 3$$

$$\bar{Y} = \frac{2+4+5+4+5}{5} = 4$$

Centered data:

- $X_c = X - \bar{X} = [-2, -1, 0, 1, 2]$
- $Y_c = Y - \bar{Y} = [-2, 0, 1, 0, 1]$

Step 2: Compute weight vector (w)

Since we have one predictor,

$$w = \frac{X_c^T Y_c}{\|X_c\|}$$

$$X_c^T Y_c = (-2)(-2) + (-1)(0) + (0)(1) + (1)(0) + (2)(1) = 4 + 0 + 0 + 0 + 2 = 6$$

$$\|X_c\|^2 = (-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2 = 4 + 1 + 0 + 1 + 4 = 10$$

So,

$$w = \frac{6}{\sqrt{10}} \approx 1.897$$

Step 3: Compute score (t)

$$t = X_c \cdot w$$

Since w is just scaling, t is proportional to X_c.
Thus,

$$t = [-2, -1, 0, 1, 2] \cdot (constant)$$

So effectively, **t = X_c** (for one variable case).

Step 4: Regression coefficient

In PLS, coefficient is:

$$\beta = \frac{t^T Y_c}{t^T X_c}$$

Compute numerator:

$$t^T Y_c = (-2)(-2) + (-1)(0) + (0)(1) + (1)(0) + (2)(1) = 6$$

Denominator:

$$t^T X_c = (-2)(-2) + (-1)(-1) + (0)(0) + (1)(1) + (2)(2) = 10$$

$$\beta = \frac{6}{10} = 0.6$$

Step 5: Intercept

$$\alpha = \bar{Y} - \beta \bar{X} = 4 - 0.6(3) = 2.2$$

✅ Final PLS Regression Equation

$$Y = 2.2 + 0.$$