Let's carefully do **all cofactor calculations step by step** for

$$A = X^T X = \begin{bmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix}.$$

Step 1: Formula

Cofactor of a_{ij}:

$$C_{ij} = (-1)^{i+j} \det(M_{ij})$$

where M_{ij} = minor matrix after removing row i and column j.

Step 2: Compute cofactors one by one

Row 1:

- C_{11}:

  Minor =
  $\begin{bmatrix} 55 & 225 \\ 225 & 979 \end{bmatrix}$
  $\det = 55(979) - 225(225) = 53,845 - 50,625 = 3,220$
  $\Rightarrow C_{11} = +3220$

- C_{12}:

  Minor =

$$\begin{bmatrix} 15 & 225 \\ 55 & 979 \end{bmatrix}$$

$\det = 15(979) - 225(55) = 14,685 - 12,375 = 2,310$

$\Rightarrow C_{12} = (-1)^{1+2}(2310) = -2310$

- C_{13}:

  Minor =

  $$\begin{bmatrix} 15 & 55 \\ 55 & 225 \end{bmatrix}$$

  $\det = 15(225) - 55(55) = 3,375 - 3,025 = 350$

  $\Rightarrow C_{13} = +350$

Row 2:

- C_{21}:

  Minor =

  $$\begin{bmatrix} 15 & 55 \\ 225 & 979 \end{bmatrix}$$

  $\det = 15(979) - 55(225) = 14,685 - 12,375 = 2,310$

  $\Rightarrow C_{21} = (-1)^{2+1}(2310) = -2310$

- C_{22}:

  Minor =

  $$\begin{bmatrix} 5 & 55 \\ 55 & 979 \end{bmatrix}$$

  $\det = 5(979) - 55(55) = 4,895 - 3,025 = 1,870$

  $\Rightarrow C_{22} = +1870$

- C_{23}:

  Minor =

  $$\begin{bmatrix} 5 & 15 \\ 55 & 225 \end{bmatrix}$$

  $\det = 5(225) - 15(55) = 1,125 - 825 = 300$

  $\Rightarrow C_{23} = (-1)^{2+3}(300) = -300$

Row 3:

- C_{31}:

  Minor =

  $$\begin{bmatrix} 15 & 55 \\ 55 & 225 \end{bmatrix}$$

  $\det = 15(225) - 55(55) = 3,375 - 3,025 = 350$

  $\Rightarrow C_{31} = +350$

- C_{32}:

  Minor =

  $$\begin{bmatrix} 5 & 55 \\ 15 & 225 \end{bmatrix}$$

$$\det = 5(225) - 55(15) = 1,125 - 825 = 300$$

$$\Rightarrow C_{32} = (-1)^{3+2}(300) = -300$$

- C_{33}:

  Minor =

  $$\begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix}$$

  $$\det = 5(55) - 15(15) = 275 - 225 = 50$$

  $$\Rightarrow C_{33} = +50$$

Step 3: Cofactor Matrix

$$C = \begin{bmatrix} 3220 & -2310 & 350 \\ -2310 & 1870 & -300 \\ 350 & -300 & 50 \end{bmatrix}$$

Step 4: Adjoint (transpose of cofactor matrix)

$$\mathrm{Adj}(A) = \begin{bmatrix} 3220 & -2310 & 350 \\ -2310 & 1870 & -300 \\ 350 & -300 & 50 \end{bmatrix}$$

✅ This is the **correct adjoint** of $X^T X$.

$\det(A)=5(3220)+15(-2310)+55(350)=16100-34650+19250=700$

Inverse (exact)

$$A^{-1} = (X^T X)^{-1} = \frac{1}{700}\mathrm{Adj}(X^T X) = \begin{bmatrix} \frac{23}{5} & -\frac{33}{10} & \frac{1}{2} \\ -\frac{33}{10} & \frac{187}{70} & -\frac{3}{7} \\ \frac{1}{2} & -\frac{3}{7} & \frac{1}{14} \end{bmatrix} \approx \begin{bmatrix} 4.6 & -3.3 & 0 \\ -3.3 & 2.6714286 & -0.42 \\ 0.5 & -0.4285714 & 0.071 \end{bmatrix}$$

Given $X^T Y = \begin{bmatrix} 28 \\ 106 \\ 446 \end{bmatrix}$ (from your image).

Compute

$$\mathbf{a} = (X^T X)^{-1}(X^T Y).$$

Exact result (fractions):

$$\mathbf{a} = \begin{bmatrix} 2 \\ -\frac{13}{35} \\ \frac{3}{7} \end{bmatrix}$$

Decimal approximation:

$$\mathbf{a} \approx \begin{bmatrix} 2.0 \\ -0.37142857 \\ 0.42857143 \end{bmatrix}$$

So the final computed vector $A = (X^T X)^{-1} X^T Y$ is $\boxed{[2,\ -13/35,\ 3/7]^T}$.

We found

$$\mathbf{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{13}{35} \\ \frac{3}{7} \end{bmatrix}$$

Final Regression Equation

$$Y = a_0 + a_1 X + a_2 X^2$$

Substituting the coefficients:

$$Y = 2 - \frac{13}{35} X + \frac{3}{7} X^2$$

✅ **Final Answer:**

$$\boxed{Y = 2 - 0.3714X + 0.4286X^2}$$